RESEARCH DEPARTMENT

THE SPECIFICATION OF THE REFLECTION COEFFICIENT OF A F.M. AERIAL

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SUMMARY

If an aerial radiating f.m. transmissions is not matched to its feeder, amplitude modulation is introduced and the a.f. signal suffers non-linear distortion. The distortion is most marked if the receiver lacks a limiter. These effects are examined in this report and a limit for the reflection coefficient at the aerial is proposed. This constitutes a considerable relaxation of the specification of the Wrotham aerial.

SYMBOLS

$A(t)$	Instantaneous amplitude of radiated signal.
$B \sin \omega t$	Single-tone modulation expressed in terms of the instantaneous angular frequency deviation (radians/sec.).
B_n	Amplitude of n^{th} harmonic of single-tone signal after distortion.
$F(t)$	Instantaneous angular frequency deviation (radians/sec.) at the transmitter - general case.
$F_1(t)$	Instantaneous angular frequency deviation (radians/sec.) of the radiation from the aerial. Proportional to a.f. output of receiver type (a).
$F_2(t)$	A.F. output of receiver type (b).
$J_n(x)$	Bessel function of the first kind, order n, argument x.
T	Time constant of de-emphasis (seconds).
w	Loss in a single traversal of the feeder and combining filter (db).
μ	Amplitude of the delayed signal relative to that of the primary signal.
ρ	Reflection coefficient at the aerial.
ρ'	Reflection coefficient at the bottom of the feeder when this is terminated in a matched load.
τ	Time delay of delayed signal (seconds).
φ	Phase advance of the delayed signal, relative to the primary signal, at the carrier frequency.

The following numerical values are used, except where otherwise stated:

B	$4{\cdot}71 \times 10^5$ radians/sec. (equivalent to 75 kc/s).
τ	$2{\cdot}03 \times 10^{-6}$ sec. (Time for traversal of 1,000 feet (305 metres) of air-spaced feeder, and return).
$B\tau$	$0{\cdot}96$ radians ($\equiv 55^\circ$).
T	5×10^{-5} seconds.

1. INTRODUCTION.

An aerial for multi-channel v.h.f. broadcasting must operate over a wide band of frequencies, since the minimum separation between channels is set by receiver design considerations. If three transmissions are to be radiated at frequencies spaced 2·2 Mc/s, the bandwidth of the aerial must be at least 4·6 Mc/s; but 7 Mc/s would be preferable, so that the entire f.m. band (88–95 Mc/s) could be covered without adjustment.

The chief difficulty in achieving the required bandwidth is in maintaining the aerial impedance within sufficiently close limits, so that the standing-wave ratio will not be less than the minimum permitted by the specification. At Wrotham the standing-wave ratio is not less than 0·9 over a band of 5·5 Mc/s; but this result was achieved only at great expense, of which development work and adjustment accounted for an appreciable proportion. In order to avoid unnecessary expenditure at future v.h.f. stations, it is desirable to know the lowest standing-wave ratio that can be permitted without sacrifice of programme quality.

A low standing-wave ratio may degrade the performance in four ways:

i. By causing additional loss in the transmission line.

ii. By restricting the power-handling capacity of the transmission line and distribution feeders.

iii. By causing distortion.

iv. By giving rise to amplitude modulation. This will increase the distortion if the receiver has no limiter, and may be objectionable on other grounds.

The additional loss is never likely to be an important factor; for example, a decrease in standing-wave ratio from unity to 0·75 would cause a small loss to increase by about 4%. For 1,000 feet of Marconi 5 in. transmission line, this increase would be only 0·035 db. The power-handling capacity, whether determined by the maximum voltage or by local heating, is directly proportional to the standing-wave ratio (expressed as a number less than unity). This consideration might in some cases influence the closeness of the matching required.

The purpose of this report is to summarise the effect of standing waves in the transmission line on the quality of the received programmes, and to propose a specification for the matching of transmitting aerials. Since the theoretical analysis is straightforward, only the results will be given.

2. THE EFFECT OF MISMATCH ON THE R.F. SIGNAL.

If the aerial is imperfectly matched to the feeder, waves reaching it will be reflected back towards the transmitter. If the source impedance of the transmitter were equal to the characteristic impedance of the line, the reflected waves would be entirely absorbed, but in practice this condition is not satisfied, so that the waves undergo a second reflection at the transmitter and again reach the aerial. It is difficult to predict the nature of this second reflection - particularly in the case of Class B or Class C output stages, which do not behave as linear networks - but it will be assumed that the reflection coefficient at the transmitter is about 0·85. In addition to the 15% loss in amplitude due to reflection at the transmitter, the reflected waves will be attenuated in two additional traversals of the feeder.

In practice the reflection coefficients at both aerial and transmitter will be approximately constant in magnitude and phase over the frequency band occupied by any one f.m. channel. The transmitted signal may therefore be regarded as consisting of two components of constant amplitude: the primary signal, which will have traversed the feeder once, and the delayed signal, which will have traversed the feeder three times. Since, as will appear later, the reflection coefficient must be made small, waves reflected more than once at the aerial need not be considered. If μ is the ratio of the amplitude of the delayed signal to that of the primary signal, μ^2 will be neglected.

The phase difference between the direct and reflected waves will be proportional to the frequency, varying as the frequency is modulated; for example, if the length of the line is 1,000 feet (305 metres), a frequency deviation of ± 75 kc/s will cause the phase difference to vary by $\pm 55^\circ$. The resultant will in consequence be amplitude-modulated, and its instantaneous frequency will contain unwanted components.

The combining filter will add to the effective length of the feeder. The relevant parameter is the group delay $d\varphi/d\omega$, where φ is the phase shift at an angular frequency ω. Measurements on the Wrotham filter indicated a group delay equivalent to 600 feet (183 metres) of air-spaced feeder, but this filter has an unnecessarily high stop-band insertion loss. In the absence of details of filters to be used at future stations it seems reasonable to assume a group delay equivalent to 300 feet (91 metres) of air-spaced feeder.

The distortion of the frequency modulation and the depth of unwanted amplitude modulation will depend upon φ, the phase difference between the principal and delayed signals at the unmodulated carrier frequency. This phase difference could be controlled by adjusting the effective length of the feeder.

3. TYPES OF RECEIVER.

The effect upon the a.f. output of the receiver will depend upon the extent to which this responds to amplitude modulation; three cases are considered below.

a. Perfect limiter.

If the limiter is perfect the receiver will be unaffected by amplitude modulation, and may be assumed to give an output proportional to the instantaneous frequency deviation. This case corresponds to a good f.m. receiver and a strong signal.

b. Balanced discriminator but no limiter.

In the absence of a limiter, the instantaneous output of a balanced discriminator will be proportional to the product of the instantaneous amplitude and the instantaneous frequency deviation. This case would apply to a good f.m. receiver used where the field is too weak to operate the limiter, but it is possible that distortion due to reflection in the transmission line would then be masked by similar distortion due to multi-path propagation, and by noise.

c. Detuned a.m. receiver.

It is possible that, during the initial development of v.h.f. broadcasting, some listeners in areas of high field strength will use receivers without either a limiter or a balanced discriminator: e.g. a detuned a.m. receiver. The distortion would be more severe than in receivers (a) and (b), and would depend upon the relative sensitivity to frequency and amplitude modulation. Since it is not considered practicable to cater for this type of receiver it will not be considered further in this report.

4. AMPLITUDE MODULATION.

The instantaneous amplitude is proportional to:

$$A(t) = 1 + \mu\cos\left\{\tau F(t) - \varphi\right\} \tag{1}$$

For a peak frequency deviation of 75 kc/s, and a feeder length of 1,000 feet (305 metres), the maximum value of $\tau F(t)$ is 0·96 radians (55°). The greatest depth of amplitude modulation occurs if φ is equal to $\pm\pi/2$, and is then equal to 82 μ%. By adjusting the feeder length to make φ equal to 0 or π, the depth could be reduced to 21 μ%.

5. DISTORTION WITH RECEIVER (a) – Perfect limiter.

5.1 The General Case.

Pre-emphasis and de-emphasis will be ignored in the first instance.

The distorted output of an ideal receiver of type (a) may be shown to be given by:

$$F_1(t) = F(t) - \mu\frac{d}{dt}\left[\sin\left\{\tau F(t) - \varphi\right\}\right] \qquad (2)$$

The distortion could have been produced by passing the undistorted a.f. signal through a non-linear network and a differentiating circuit, and combining the output with the undistorted signal. It is difficult to find a satisfactory quantitative assessment of this type of distortion, which is not commonly encountered in audio-frequency work. The difficulty is caused by the differentiation in (2), which results in the amplitude of the harmonics produced by the distortion of a pure tone being proportional to the frequency of the fundamental.

5.2 A Single Sinusoidal Tone.

Substituting $B \sin \omega t$ for $F(t)$ in (2), and expanding in terms of Bessel functions, it is found that the ratio of the amplitude B_n of the n^{th} harmonic to B, that of the fundamental, is:

$$B_n/B = (2\mu\,\omega n/B)\left[\sin(\varphi + n\pi/2)\right] J_n(\tau B) \qquad (3)$$

The relative amplitudes of the harmonics depend upon φ. If this phase difference is varied by changing the length of the transmission line, the amplitudes will vary in a cyclical manner, odd harmonics being strongest when the even harmonics are weakest, and vice versa.

Table I summarises the numerical results for 1,000 feet of feeder and a peak frequency deviation of 75 kc/s. The frequency of the a.f. tone has been taken to be 1,000 c/s; for other frequencies the percentage harmonic amplitudes are altered in proportion. The percentages indicate the maximum values of B_n/B for an arbitrary phase difference φ, although these maximum values could not in fact be achieved by odd and even harmonics in the same case. If the feeder length could be adjusted, the best results would be obtained by making φ equal to 0 or π; the even harmonics would then be suppressed.

TABLE I

Harmonic Distortion of a 1,000 c/s Sinusoidal Tone
(Limiter. No de-emphasis)
(For other fundamental frequencies the percentages are increased in proportion to the fundamental frequency)

μ	n			
	2	3	4	5
0·025	0·014%	0·003%	0·001%	0·000%
0·05	0·028%	0·007%	0·001%	0·000%
0·10	0·056%	0·014%	0·002%	0·000%
0·15	0·084%	0·020%	0·003%	0·000%
0·20	0·112%	0·027%	0·004%	0·001%

In a.f. amplifiers the percentage harmonic distortion is usually independent of the fundamental frequency. It is then possible to specify the degree of distortion in terms of the amplitudes of the harmonics, and from experience it is known whether a given set of amplitudes will give acceptable quality. In order to apply this experience to the present problem it is necessary to decide on the typical fundamental frequency. In the absence of a better method of doing this, the following procedure will be adopted.

The relative amplitude of each harmonic is computed on the assumption that its frequency is 10 kc/s. Thus in computing the relative amplitude B_n/B of the n^{th} harmonic the fundamental frequency is taken as 10,000/n c/s. It will be assumed that the subjective effect of the distortion is not more severe than that produced by a simple non-linear network giving the same set of relative amplitudes at all frequencies. This procedure is based on the assumption that the insensitivity of the ear at frequencies above 10 kc/s will at least compensate for the underestimate of distortion products having higher frequencies. If, therefore, the non-linear distortion is assessed in terms of the amplitudes of the harmonics listed in Table II, the assessment will not be optimistic, but may be pessimistic.

TABLE II

Relative Amplitude of the n^{th} Harmonic of a Pure Tone of Frequency 10,000/n c/s

(Limiter. No de-emphasis)

μ	n			
	2	3	4	5
0·025	0·070%	0·011%	0·001%	0·000%
0·05	0·14%	0·023%	0·003%	0·000%
0·10	0·28%	0·045%	0·005%	0·001%
0·15	0·42%	0·068%	0·008%	0·001%
0·20	0·56%	0·091%	0·011%	0·001%

5.3 Effect of Pre-emphasis and De-emphasis.

Pre-emphasis cannot alter the figures given in Tables I and II, since these results are based on a given peak frequency deviation: 75 kc/s. It may nevertheless modify the effect of non-linear distortion on the subjective quality, since it changes the spectrum of the modulating signal. Since this effect is not readily evaluated it will be ignored.

De-emphasis attenuates the higher audio frequencies in proportion to a factor

$$1/\sqrt{(1 + \omega^2T^2)}$$

where T is the time constant of the de-emphasis network, 50 μs in the B.B.C. Frequencies below 2 kc/s will not be affected greatly; above 5 kc/s the attenuation will increase by about 6 db per octave. The effect will be to compensate partially for the differentiation in Equation 2, particularly at the higher frequencies, so that the non-linear distortion will resemble more nearly that produced by a simple non-linear network. Thus the dependence of the relative amplitude of the harmonics, produced by the distortion of a pure tone, on the fundamental frequency will be reduced but not eliminated.

Following the procedure adopted in Section 5.2, the relative amplitude of each harmonic will be computed on the assumption that its frequency is 10 kc/s. To take account of the effect of de-emphasis on both fundamental and harmonic frequencies, the percentages in Table II should be multiplied by

$$\sqrt{\left\{(1 + \omega^2 T^2)/(1 + n^2\omega^2 T^2)\right\}}$$

where $n\omega = 2\pi \times 10^4$ radians/sec. and $T = 5 \times 10^{-5}$ sec. The result is contained in Table III.

TABLE III

Relative Amplitude of the n^{th} Harmonic of a Pure Tone of Frequency 10,000/n c/s

(Limiter. 50 μs de-emphasis)

μ	n			
	2	3	4	5
0·025	0·040%	0·005%	0·001%	0·000%
0·05	0·079%	0·010%	0·001%	0·000%
0·10	0·16%	0·020%	0·002%	0·000%
0·15	0·24%	0·030%	0·003%	0·000%
0·20	0·32%	0·040%	0·004%	0·000%

6. DISTORTION IN RECEIVER (b) – No limiter.

6.1 The General Case.

Again pre-emphasis and de-emphasis will be ignored in the first instance.

Additional distortion will arise from the effective multiplication in the discriminator of the instantaneous frequency deviation by the instantaneous amplitude. It will be shown that this distortion is much more severe than that associated with distortion of the instantaneous frequency deviation, which was considered in Section 5, and which will still occur. For this reason distortion of the instantaneous frequency deviation will be neglected. The resultant a.f. signal $F_2(t)$ is therefore

Obtained by multiplying together the right-hand sides of Equations *1* and *2*, neglecting the second term in Equation *2*. Thus:

$$F_2(t) = F(t) + \mu F(t)\cos\left\{\tau F(t) - \varphi\right\} \qquad (4)$$

This is the more familiar type of non-linear distortion such as would be produced by a simple non-linear network.

6.2 A Single Sinusoidal Tone.

Substituting B sin ωt for F(t) in *(2)* and expanding in terms of Bessel functions it is found that the relative amplitude B_n/B of the n^{th} harmonic is given by:

$$B_n/B = \mu\left\{J_{n-1}(\tau B) - J_{n+1}(\tau B)\right\}\left|\sin(\varphi + n\pi/2)\right| \qquad (5)$$

$J_{n+1}(\tau B)$ may be neglected in comparison with $J_{n-1}(\tau B)$. Thus:

$$B_n/B \doteqdot \mu J_{n-1}(\tau B)\left|\sin(\varphi + n\pi/2)\right| \qquad (6)$$

B_n/B, expressed as a percentage, is tabulated against n and μ in Table IV.

TABLE IV

Relative Amplitude of the n^{th} Harmonic of a Pure Tone

(No Limiter)

μ	n			
	2	3	4	5
0·025	1·1%	C·26%	0·042%	0·005%
0·05	2·1%	0·53%	0·085%	0·010%
0·10	4·3%	1·06%	0·17%	0·021%
0·15	6·4%	1·6%	0·25%	0·031%
0·20	8·5%	2·1%	0·34%	0·041%

As in Tables I-III, the percentages given in Table IV are maximum values for an arbitrary value of φ; in fact the odd and even harmonics could not attain their maximum amplitudes simultaneously. In both receivers, (a) and (b), the second harmonic is of greatest importance. This, with the other even harmonics, is a maximum when $\varphi = \pm\pi/2$, the condition giving the greatest depth of amplitude modulation.

6.3 Effect of De-emphasis.

Table IV will be unaffected by de-emphasis provided that the fundamental frequency is such that the frequencies of the harmonics are below about 2 kc/s. For higher fundamental frequencies the relative amplitudes of the harmonics will be reduced. It follows that if the signal is assumed to be distorted by a simple non-linear network giving the distortion characterised by Table IV, the assessment of the quality of reproduction will tend to be pessimistic.

7. SPECIFICATION OF THE AERIAL.

Since the properties of an aerial covering a band of at least 4·6 Mc/s will be approximately constant in the band occupied by a single f.m. channel, distortion can be caused only by reflection. To avoid distortion it is sufficient to specify the reflection coefficient.

7.1 Maximum Amplitude of Delayed Signal - 1,000 feet of Feeder.

In determining the maximum permissible value of μ, it is sufficient to consider Tables III and IV, which are applicable respectively to receivers with and without limiters. It will be assumed that the phase angle φ, which is determined by the feeder length, will not be controlled, and will have the least favourable value. This is $\pm\pi/2$, the value giving the maximum amplitude of the even harmonics. The second harmonic then predominates, other even harmonics falling off rapidly with increasing order. Even if the "annoyance" is assessed by weighting harmonics according to the square of the order (this procedure was provisionally recommended in Research Report No. L.023*) the conclusions would not be affected appreciably.

It is recommended that the aerial should be specified so that μ should not exceed 0·075. The maximum depth of amplitude modulation will then be 6%. The relative amplitude of the second harmonic in the output of an ideal receiver with a limiter will be 0·12% (interpolated from Table III). This is a negligibly small amount of distortion. If there is no limiter the relative amplitude of the second harmonic will be 3·2% (interpolated from Table IV). This degree of distortion would be perceptible only to the most discriminating listener, who might be expected to provide himself with a good receiver having a limiter.

7.2 Feeders of Different Effective Lengths.

Up to this point the argument has been simplified by considering only an effective feeder length of 1,000 feet (it must be remembered that this includes an allowance of about 300 feet for group delay in the filter).

* Research Report No. L.023, Serial No. 1949/30... "An Investigation into Non-Linear Distortion: First Interim Report". The procedure was to multiply the amplitude of each harmonic by the square of its order, sum all the terms so produced, and divide the result by 4.

For other lengths it is proposed to vary μ so as to maintain the maximum depth of amplitude modulation at 6%. Table V shows the value of μ for different lengths, and indicates the relative amplitude of the second harmonic that would result. The latter figures correspond to those given in Tables III and IV for an effective length of 1,000 feet.

TABLE V

Recommendations for Feeders of Various Effective Lengths, Based on 6% a.m.

Effective length of feeder in feet	Max. value of μ	Relative amplitude of second harmonic	
		Receiver (a) (with limiter)	Receiver (b) (no limiter)
400	0·165	0·05%	3·1%
600	0·113	0·07%	3·1%
800	0·089	0·09%	3·2%
1,000	0·075	0·12%	3·2%
1,200	0·067	0·15%	3·3%
1,400	0·063	0·18%	3·4%

The shorter feeders would not apply to a normal station with a combining filter, since the effective length of this is 300 feet. It is, however, possible that at some very-low-power or temporary installations separate aerials might be used to avoid the need for a filter.

7.3 The Maximum Reflection Coefficient.

The delayed signal suffers attenuation in two additional traversals of the feeder and the combining filter, and is further attenuated by imperfect reflection at the transmitter. There is some doubt as to the magnitude of the reflection coefficient at the transmitter, but it will be assumed to be 0·85.

Allowance must be made for reflection in the feeder, though feeders used hitherto have been so uniform that their contribution to the delayed signal could be neglected. It is possible that a less uniform feeder would be considered in the future if a saving in cost would thereby result. For the sake of simplicity two pessimistic assumptions will be made. Firstly, it will be assumed that all the reflection occurs at the top of the feeder, giving rise to a delayed signal with the maximum delay. Secondly, it will be assumed that any reflection in the feeder will reinforce the reflection at the aerial.

The considerations discussed above lead to the following conclusion. If μ is the maximum amplitude of the delayed signal (from Table V), ρ is the reflection coefficient at the aerial, ρ' that at the bottom of the feeder when this

is terminated in a matched load, and w is the loss in decibels for a *single* traversal of the feeder and filter, then

$$10^{-w/10}\rho + \rho' \quad \text{must not exceed} \quad \mu/0{\cdot}85 \qquad (7)$$

In assessing ρ', mismatch at the combining filter and at points nearer to the transmitter may be neglected in view of the short time delay for the resulting delayed signal. The fact that reflections in the feeder will undergo attenuation in the filter may also be neglected.

As an example we shall consider the Wrotham aerial. The length of the main feeder is 730 feet. To this should be added about 100 feet for distribution feeders and feeders between the filter and the transmitters, and 600 feet for group delay in the filter (it is expected that the group delay will be about 300 feet at future stations). Hence the effective length is about 1,400 feet. According to Table V, μ should not exceed 0·063.

The loss in the feeders and combining filter is about 1 db. The feeder itself has a reflection coefficient of about 1% when terminated in a matched load. Inserting these values in Equation 7, it is found that the maximum value of ρ is 0·08. In fact the specified limit was 0·05.

8. CONCLUSIONS.

The maximum level of delayed signal (μ) is given in Table V for various effective feeder lengths. The effective length includes about 300 feet to take account of group delay in the combining filter. The maximum reflection coefficient at the aerial is given in terms of μ in Equation 7, which takes account of loss and reflection in the feeder, and imperfect reflection at the transmitter. The proposed specification constitutes a considerable relaxation of that framed for Wrotham, but it is possible that at some stations power-handling considerations may render it impracticable to take full advantage of this relaxation.

It would be possible to relax the specification of the aerial still further if the phase of the delayed signal, relative to that of the primary signal, could be controlled, e.g. by adjusting the length of the feeder between each transmitter and the combining filter. By making the phase difference zero or π, second-harmonic distortion would be suppressed, and the depth of amplitude modulation would be reduced. It does not appear to be worth while to do this at most stations, but controlled feeder lengths might be of value in facilitating a make-shift arrangement, where a low-power transmission is required at an existing site and the mast cannot accommodate a wide-band aerial.